Propagation of Love type wave in piezoelectric layer overlying non-homogeneous half-space

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Abstract

The present paper investigates, the mathematical modelling of the existence of Love type waves in a piezoelectric layer overlying a non-homogeneous half-space. Piezoelectric layer is considered for two different cases one is electrical open circuit and another one is electrical short circuit. The general dispersion equation has been derived for both the cases. As a special case dispersion equation has been obtained when the half-space is homogeneous medium. The velocities of Love waves have been calculated numerically as a function of wave number kh. The effect of non-homogeneity and dielectric constant are illustrated by graphs in both electrically open and electrically short circuit cases. All the figures show that phase velocity decreases with the increases of wave number kh. Using MATLAB software, graphical user interface (GUI) has been developed to generalize the effect of parameters discussed. The results can be used to understand the nature of wave propagation in piezoelectric structures.

1 Introduction

Seismology is the study of earthquake and seismic wave that tells us about the structure of Earth and physics of earthquake. A seismologist is a scientist who studies earthquakes and seismic waves. The science of seismology aims simultaneously to know the infrastructure of the Earth's interior with the help of seismic wave phenomena and to study the nature of earthquake sources with ultimate gole of mitigating and eventually controlling the phenomena. If the Earth rapidly displaced at some point then energy imparted into the Earth by the source of the distortion can be transmitted in the form of elastic waves. Ewing et al. [13] gave the basic literature on the propagation of elastic waves.

A.E.H Love [22] developed a mathematical model of surface waves known as Love waves. He predicted the existence of Love waves mathematically in 1911. The problems of propagation of Love waves in the anisotropic and non-homogeneous medium have of great practical importance.

They are not only helpful in investigating the internal structure of Earth but also are very helpful in exploration of natural resources buried inside the Earths surface. These waves are propagated when the solid medium near the surface has non-homogeneous elastic properties. In Love-type waves, there is no particle motion in the vertical plane but particle motion takes place in the horizontal plane only and it is transverse to the direction of propagation. Chattopadhyay [7] derived his idea on the dispersion equation for Love wave due to irregularity in the thickness of non-homogeneous crustal layer. Chakraborty and Dey [5] have found that the propagation of Love waves in water saturated soil underlain by heterogeneous elastic medium. A detail study on seismology and plate tectonics was made by Gubbins [15]. Dey et al. [9] investigated propagation of Love waves in heterogeneous crust over a heterogeneous mantle. Love waves in a two-layered half-space was studied by Singh [30]. Abd-Alla and Ahmed [1] discussed the propagation of Love waves in a non-homogeneous orthotropic elastic layer under initial stress overlying semi-infinite medium. Wave transport for a scalar model of the Love waves was observed by Bal and Ryzhik [2]. The Influence of anisotropy on the Love waves in a self-reinforced medium had obtained by Pradhan et al. [25].

Love wave propagating in the piezoelectric materials is extensively used in sensors and transducers. The propagation of Love wave in elastic or piezoelectric materials has been investigated by many researchers. Li et al. [20] delivered his idea on the propagation of Love waves in functionally graded piezoelectric materials. Du et al. [10] investigated Propagation of Love waves in pre-stressed piezoelectric layered structures loaded with viscous liquid. Cao et al. [6] studied propagation of Love waves in a functionally graded piezoelectric material (FGPM) layered composite system. Effect of an imperfect interface on the SH wave propagating in a cylindrical piezoelectric sensor was discussed by Li and Lee [19].

Propagation of waves in piezoelectric structural layer have received considerable attention previously as exhibited by the work of Eskandari and Shodja [12], Du et al. [11], Nie et al. [23] Sharma et al. [28] [29]. Qian et al. [26] found the dispersion characteristics of transverse surface waves in piezoelectric coupled solid media with hard metal interlayer. The effect of initial stress on the propagation behavior of SH waves in piezoelectric coupled plates was introduced by Sona and Kang [32] after that in [33] they have been found the effect of shear wave propagation in a layered poroelastic structure. Li et al. [21] studied a three-layer structure model for the effect of a soft middle layer on Love waves propagating in layered piezoelectric systems. Piliposian et al. [24] formulated the shear wave propagation in periodic phononic/photonic piezoelectric medium. Theoretical validation of the existence of two transverse surface waves in piezoelectric/elastic layered structures had also assumed by Qian and Hirose [27].

Many works have been done by many researchers in the field of Love wave propagation. Some of them are Ghorai et al. [14], Gupta et al. [16], W.H. Sun et al. [31]. Gupta et al. [17] established propagation of Love waves in non-homogeneous substratum over initially stressed heterogeneous half-space. Possibility of Love wave propagation in a porous layer under the effect of linearly varying directional rigidities was obtained by Gupta et al. [18]. Chattopadhyay et al. [8] investigated dispersion of horizontally polarized shear waves in an irregular non-homogeneous self-reinforced crustal layer over a semi-infinite Self-reinforced Medium.

In this paper, we carry out an investigation of the existence of propagation of Love wave in layered piezoelectric overlying a non-homogeneous half-space. Piezoelectric sensitive layer structures having layer electromechanical coupling factors, PZT-4 metal has been widely applied. Piezoelectric layer is considered for two different cases one is electrically open circuit and another one is electrically short circuit. The influences of dielectric constant and inhomogeneity parameter for the layer and half-space are also discussed.

2 Formulation of the Problem

Consider an elastic piezoelectric layer of thickness h (where $-h \leq z \leq 0$) overlying a non-homogeneous half-space as shown in figure 1.

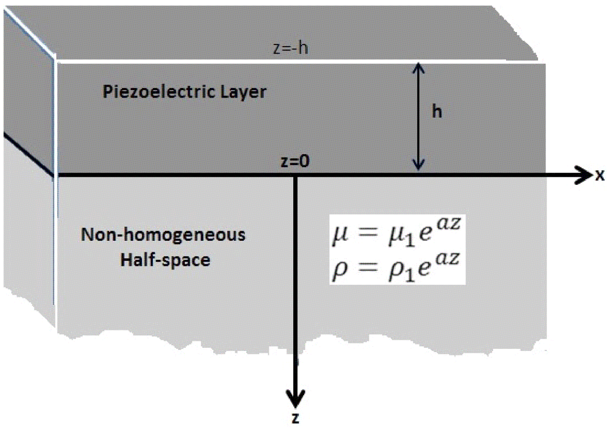

Figure 1: Geometry of the problem

The piezoelectric layer is deposited perfectly on the heterogeneous layer, which results in a surface at $z = -h$ free of external forces. The plane $z = 0$ is surface of contact of piezoelectric layer and half-space and the z-axis is vertically downwards. The x-axis is parallel to the direction of propagation of the wave. So, the non-zero field of quantities representing the motion are only function of x, z and time t.

3 Solution of Problem

3.1 Solutions of Piezoelectric Layer

A piezoelectric structure involving a thin piezoelectric layer bonded perfectly over a non-homogeneous elastic half-space is illustrated in figure 1. The piezoelectric material is polarized along the x-direction. The constitutive equations of the piezoelectric medium can be written as Li et al. [21]

$$\sigma_{ij} = c_{ij,kl}S_{kl} - e_{k,ij}E_k \quad (1)$$

$$D_j = e_{j,kl}S_{kl} + \varepsilon_{jk}E_k \quad (2)$$

where σ_{ij} and S_{kl} are the stress and strain tensors, E_k is the electrical potential field, D_j is the electrical displacement and c_{ijkl}, e_{kij} and ε_{jk} are the elastic, piezoelectric and dielectric coefficients respectively.

Now, in the problem the waves are propagating along x-direction and the material properties of piezoelectric layer vary continuously along the direction of thickness. Therefore the motion equation and the electrical displacement equilibrium equation are given by

$$\sigma_{ij,j} = \rho \ddot{v}_i, \tag{3}$$

$$D_{i,i} = 0 \tag{4}$$

where v_i is the component of mechanical displacement in the i-th direction and ρ is the mass density. The propagation of Love waves may be represented by displacement components and electrical potential function as

$$\left.\begin{array}{c} u = 0 = w, \;\; v = v_1(x,z,t) \\ \phi = \phi(x,z,t) \end{array}\right\} \tag{5}$$

where u, v and w are the mechanical displacement components in the x, y and z direction, respectively, and ϕ is the electrical potential function.
Now, the relation between the mechanical displacement and the strain components is as follows

$$S_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i}) \tag{6}$$

According to the quasi-static Maxwell's equation, the relation between the electrical intensity and the electrical potential is

$$E_i = -\frac{\partial \phi}{\partial x_i} \tag{7}$$

Typically, for the transversely isotropic piezoelectric, equations (1) and (2), can be expressed in the rectangular form as

$$\left.\begin{array}{l} \sigma_x = c_{11}S_x + c_{12}S_y + c_{13}S_z - e_{31}E_z \\ \sigma_y = c_{12}S_x + c_{11}S_y + c_{13}S_z - e_{31}E_z \\ \sigma_z = c_{13}S_x + c_{13}S_y + c_{33}S_z - e_{31}E_z \\ \sigma_{xz} = (c_{11} - c_{12})\frac{S_{xz}}{2} \\ \sigma_{yz} = c_{44}S_{yz} - e_{15}E_z \\ \sigma_{xy} = c_{44}S_{xy} - e_{15}E_x \\ D_x = e_{15}S_{xy} + \varepsilon_{11}E_x \\ D_z = e_{15}S_{zy} + \varepsilon_{11}E_z \\ D_y = e_{31}S_x + e_{31}S_z + e_{31}S_y + \varepsilon_{33}E_y \end{array}\right\} \tag{8}$$

Now, substituting equation (5) into equations (6) and (7), again substituting the modified equations (6) and (7), into equation (8), using new equations into (3) and (4), we can obtained the governing equations for the mechanical displacements and the electrical potential followed by Bleustein [3]

$$\left.\begin{array}{l} c_{44} \bigtriangledown^2 v_1 + e_{15} \bigtriangledown^2 \phi = \rho \ddot{v}_1 \\ e_{15} \bigtriangledown^2 v_1 - \varepsilon_{11} \bigtriangledown^2 \phi = 0 \end{array}\right\} \tag{9}$$

where v_1 and ϕ denote the mechanical displacement and electrical potential function in the piezoelectric layer and c_{44}, e_{15}, ε_{11} and ρ are the elastic, piezoelectric, dielectric constants and mass density respectively, $\bigtriangledown^2$ is the two dimensional Laplacian and t is time.

Now, the equation (9) takes the form as

$$\bigtriangledown^2 v_1 - \frac{1}{c_0^2}\ddot{v}_1 = 0 \tag{10}$$

$$\bigtriangledown^2 \phi - \frac{1}{c_0^2}\left(\frac{e_{15}}{\varepsilon_{11}}\right)\ddot{v}_1 = 0 \tag{11}$$

where $c_0 = \sqrt{\frac{\bar{c}_{44}}{\rho}}$ and $\bar{c}_{44} = \left(c_{44} + \frac{e_{15}^2}{\varepsilon_{11}}\right)$, c_0 is the bulk-shear-wave velocity in the transversely piezoelectric layer.
We assume the solutions of the equations (10) and (11) in the form of

$$v_1(x,z,t) = V_1(z)e^{ik(x-ct)}$$
$$\phi(x,z,t) = \phi_1(z)e^{ik(x-ct)}$$

where k is the wave number, c is the phase velocity, $i = \sqrt{-1}$ and $V_1(z)$ be the solution of the equation given by

$$\frac{d^2V_1(z)}{dz^2} + k^2b^2V_1(z) = 0 \tag{12}$$

where $b = \sqrt{\left(\frac{c^2}{c_0^2} - 1\right)}$, $\frac{c}{c_0}$ is the phase velocity ratio
and $\phi_1(z)$ be the solution of the equation

$$\frac{d^2\phi_1(z)}{dz^2} - k^2\phi_1(z) + \frac{1}{c_0^2}\left(\frac{e_{15}}{\varepsilon_{11}}\right)\{A_1 sin(kbz) + A_2 cos(kbz)\}\,k^2c^2 = 0 \tag{13}$$

where A_1 and A_2 are arbitrary constants.
With out loss of generality, it is assumed that the Love waves propagated along the positive direction of the x-axis, so the solutions of the equations (10) and (11) can be written as

$$v_1(x,z,t) = \{A_1 sin(kbz) + A_2 cos(kbz)\}\,e^{ik(x-ct)} \tag{14}$$

$$\phi(x,z,t) = \left\{\frac{e_{15}}{\varepsilon_{11}}\{A_1 sin(bkz) + A_2 cos(bkz)\} + A_3e^{-kz} + A_4e^{kz}\right\}e^{ik(x-ct)} \tag{15}$$

where A_3 and A_4 are arbitrary constants.
The stress component and electric displacement of piezoelectric layer used for the boundary and continuity conditions are

$$\sigma_{yz} = \left[kbA_1\bar{c}_{44}cos(bkz) - kbA_2\bar{c}_{44}sin(bkz) + ke_{15}\left\{-A_3e^{-kz} + A_4e^{kz}\right\}\right]e^{ik(x-ct)} \tag{16}$$

$$D_z = \varepsilon_{11}k\left\{A_3e^{-kz} - A_4e^{kz}\right\}e^{ik(x-ct)} \tag{17}$$

3.2 Solution for the half-space

The lower semi-infinite medium is consider as non-homogeneous with exponential variation in density and rigidity i.e., $\mu = \mu_1 e^{az}$ and $\rho = \rho_1 e^{az}$, where a is inhomogeneity parameter. Let u_2, v_2 and w_2 be the displacements along x, y and z axis respectively. Here, the propagation of horizontally polarized surface waves of Love type are propagating along x-direction. So the displacement components are $u = 0 = w$ and $v = v_2(x, z, t)$.

Therefore, applying the Love wave components conditions the equation of motion for a non-homogeneous elastic solid in the absence of body forces can be written from Biot [4] as

$$\frac{\partial}{\partial x}\left(\mu_1 e^{az}\frac{\partial v_2}{\partial x}\right) + \frac{\partial}{\partial z}\left(\mu_1 e^{az}\frac{\partial v_2}{\partial z}\right) = \rho_1 e^{az}\frac{\partial^2 v_2}{\partial t^2} \tag{18}$$

where μ_1 and ρ_1 are the rigidity and density of the medium respectively.

Now, the non-homogeneity in medium are taken as,

$$\mu = \mu_1 e^{az}, \qquad \rho = \rho_1 e^{az} \tag{19}$$

where μ_1 and ρ_1 are the values of rigidity μ and density ρ at $z = 0$ in the medium. Therefore for the Love-type waves, propagating along the x-direction having the displacement of the particles along the y-direction will produce only the e_{xy} and e_{yz} strain components and the other strain components will be zero. Hence the stress-strain relations gives

$$t_{xy} = 2\mu_1 e^{az}\, e_{xy}, \qquad t_{yz} = 2\mu_1 e^{az}\, e_{yz}. \tag{20}$$

Now, using the separation of variable method we substitute $v_2(x, z, t) = V_2(z)e^{ik(x-ct)}$ in (18) we obtain

$$\frac{d^2V_2}{dz^2} + a\frac{dV_2}{dz} - k^2 m_1^2 V_2 = 0 \tag{21}$$

where $m_1^2 = \left(1 - \frac{c^2}{c_1^2}\right)$, $c_1 = \sqrt{\frac{\mu_1}{\rho_1}}$ and $\omega = kc$, k is the wave number and c is the phase velocity.
Again, substituting $V_2(z) = V(z)e^{-\frac{a}{2}z}$ in (21) we get

$$\frac{d^2V}{dz} - k^2 n^2 V = 0 \tag{22}$$

where $n^2 = \left(m_1^2 + \frac{a^2}{4k^2}\right)$.
Therefore, we have

$$v_2(x, z, t) = A_5 e^{-nkz} e^{-\frac{a}{2}z} e^{ik(x-ct)} \tag{23}$$

where A_5 is a arbitrary constant.
This is the required solution of the lower non-homogeneous semi-infinite medium.

4 Boundary and continuity Conditions

The propagation of Love waves in this assumed model should satisfy the following boundary and continuity conditions:

(1) The upper surface of the piezoelectric layer problem is stress free i.e., $\sigma_{yz} = 0$ at $z = -h$

(2) At the surface contact of upper layer and lower non-homogeneous semi-infinite layer, the mechanical displacement and normal component of stress are continuous and electric potential function is displacement free i.e., $v_1 = v_2$, $\sigma_{yz} = t_{yz}$, $\phi = 0$, at $z = 0$

(3) $v_2 \to 0$ as $x \to +\infty$

The electrical conditions at the free surface can be classified into two categories, i.e.,

(i) electrically open circuit: $D_z = 0$ at $z = -h$ and

(ii) electrically short circuit: $\phi = 0$ at $z = -h$,
based on the fact that the space above the piezoelectric layer is air and its permitting is much less than that of the piezoelectric material.

5 Dispersion relations

Using the above boundary conditions we get the relations from equations (14),(15), (16), (17) and (23) as follows

$$A_1 b\bar{c}_{44} cos(bkh) + A_2 b\bar{c}_{44} sin(bkh) - e_{15} A_3 \, e^{kh} + A_4 e_{15} \, e^{-kh} = 0 \tag{24}$$

$$A_2 - A_5 = 0 \tag{25}$$

$$b\bar{c}_{44} A_1 - e_{15} A_3 + e_{15} A_4 + \mu_1 A_5 \left(n + \frac{a}{2k}\right) = 0 \tag{26}$$

$$\frac{e_{15}}{\varepsilon_{11}} A_2 + A_3 + A_4 = 0 \tag{27}$$

$$A_3 e^{kh} - A_4 e^{-kh} = 0 \tag{28}$$

$$\frac{e_{15}}{\varepsilon_{11}} \left\{-A_1 sin(bkh) + A_2 cos(bkh)\right\} + A_3 e^{kh} + A_4 e^{-kh} = 0 \tag{29}$$

5.1 Dispersion relation for case of electrically open circuit

For this case eliminating A_i $(i = 1, 2, ...5)$, from the equations (24) to (28), the frequency equation for Love waves is obtained as

$$\begin{vmatrix} b\bar{c}_{44}cos(bkh) & b\bar{c}_{44}sin(bkh) & -e_{15}e^{kh} & e_{15}e^{-kh} & 0 \\ 0 & 1 & 0 & 0 & -1 \\ b\bar{c}_{44} & 0 & -e_{15} & e_{15} & \mu_1\left(n+\frac{a}{2k}\right) \\ 0 & \frac{e_{15}}{\varepsilon_{11}} & 1 & 1 & 0 \\ 0 & 0 & e^{kh} & -e^{-kh} & 0 \end{vmatrix} = 0$$

this implied that

$$\mu_1\left(n+\frac{a}{2k}\right)cos(bkh)\left(e^{kh}+e^{-kh}\right)+e^{kh}\left[\frac{e_{15}^2}{\varepsilon_{11}}e^{-kh}-\frac{e_{15}^2}{\varepsilon_{11}}cos(bkh)\right.$$
$$\left.-\quad bc_{44}sin(bkh)\right]-e^{-kh}\left[\frac{e_{15}^2}{\varepsilon_{11}}e^{kh}+\frac{e_{15}^2}{\varepsilon_{11}}cos(bkh)+bc_{44}sin(bkh)\right]=0$$

which takes the form as

$$b\bar{c}_{44}tan(bkh)-\mu_1\left(n+\frac{a}{2k}\right)+\frac{e_{15}^2}{\varepsilon_{11}}=0 \tag{30}$$

This is the dispersion equation of Love-type wave in a electrically open circuit piezoelectric layer over a non-homogeneous half-space.

5.2 Dispersion relation for case of electrically short circuit

Eliminating the arbitrary constants A_i, $(i = 1, 2, ...5)$, from the equations (24) to (27) and (29) the frequency equation for Love-type waves in this case is obtained as

$$b\bar{c}_{44}\mu_1\left(n+\frac{a}{2k}\right)cos(bkh)\left(e^{-kh}-e^{kh}\right)+\mu_1\left(n+\frac{a}{2k}\right)\frac{e_{15}^2}{\varepsilon_{11}}sin(bkh)\left(e^{-kh}\right.$$
$$\left.+e^{kh}\right)-b^2\bar{c}_{44}^2sin(bkh)\left(e^{-kh}-e^{kh}\right)+b\bar{c}_{44}\frac{e_{15}^2}{\varepsilon_{11}}e^{kh}\left(cos(bkh)-e^{-kh}\right)$$
$$-b\bar{c}_{44}\frac{e_{15}^2}{\varepsilon_{11}}e^{-kh}\left(e^{kh}-cos(bkh)\right)+b\bar{c}_{44}\frac{e_{15}^2}{\varepsilon_{11}}cos(bkh)\left(e^{-kh}-cos(bkh)\right)$$
$$-2b\bar{c}_{44}\frac{e_{15}^2}{\varepsilon_{11}}sin^2(bkh)+\left(\frac{e_{15}^2}{\varepsilon_{11}}\right)^2sin(bkh)\left(e^{-kh}-e^{kh}\right)+b\bar{c}_{44}\frac{e_{15}^2}{\varepsilon_{11}}cos(bkh)$$
$$\times\left(e^{kh}-cos(bkh)\right)=0$$

this implied that

$$-b\bar{c}_{44}\mu_1\left(n+\frac{a}{2k}\right)tanh(kh)+\mu_1\left(n+\frac{a}{2k}\right)\frac{e_{15}^2}{\varepsilon_{11}}tan(bkh)+b^2\bar{c}_{44}^2tan(bkh)$$
$$\times tanh(kh)+2b\bar{c}_{44}\frac{e_{15}^2}{\varepsilon_{11}}-4b\bar{c}_{44}\frac{e_{15}^2}{\varepsilon_{11}}\frac{1}{cos(bkh)\left(e^{kh}+e^{-kh}\right)}$$
$$+\left(\frac{e_{15}^2}{\varepsilon_{11}}\right)^2tan(bkh)tanh(kh)=0$$

which takes the form

$$\left[b^2\bar{c}_{44}^2 + \left(\frac{e_{15}^2}{\varepsilon_{11}}\right)^2\right] tan(bkh)tanh(kh) + \mu_1\left(n + \frac{a}{2k}\right)\left[\frac{e_{15}^2}{\varepsilon_{11}} tan(bkh)\right.$$
$$\left. - b\bar{c}_{44} tanh(kh)\right] + 2b\bar{c}_{44}\frac{e_{15}^2}{\varepsilon_{11}}\left(1 - \frac{2}{cos(bkh)cosh(kh)}\right) = 0 \qquad (31)$$

This is the dispersion equation of Love-type wave in a electrically short circuit piezoelectric layer over non-homogeneous half-space.

6 Particular cases

6.1 Case I

If we neglect the inhomogeneity parameter in the half-space that is $\frac{a}{2k} = 0$ then the dispersion equations (30) and (33) reduces to the equations, in both cases as

$$tan\left[kh\sqrt{\left(\frac{c^2}{c_0^2} - 1\right)}\right] = \frac{\mu_1\sqrt{\left(1 - \frac{c^2}{c_1^2}\right)} - \frac{e_{15}^2}{\varepsilon_{11}}}{\bar{c}_{44}\sqrt{\left(\frac{c^2}{c_0^2} - 1\right)}} \qquad (32)$$

and

$$\left[b^2\bar{c}_{44}^2 + \left(\frac{e_{15}^2}{\varepsilon_{11}}\right)^2\right] tan(bkh)tanh(kh) + \mu_1 n\left[\frac{e_{15}^2}{\varepsilon_{11}} tan(bkh)\right.$$
$$\left. - b\bar{c}_{44} tanh(kh)\right] + 2b\bar{c}_{44}\frac{e_{15}^2}{\varepsilon_{11}}\left(1 - \frac{2}{cos(bkh)cosh(kh)}\right) = 0 \qquad (33)$$

where $n = \sqrt{1 - \frac{c^2}{c_1^2}}$ and $b = \sqrt{\frac{c^2}{c_0^2} - 1}$
Equation (34) and (33) are the dispersion equations of Love-type wave in both electrically open and short circuit piezoelectric medium respectively over non-homogeneous half-space.

6.2 Case II

If we neglect piezoelectric constant i.e., $e_{15} = 0$ from upper layer and inhomogeneity from half-space, then the dispersion equations (30) and (33) reduces to the general equation of Love wave (predicted by A.E.H. Love, 1911)

$$tan\left[kh\sqrt{\left(\frac{c^2}{c_0^2} - 1\right)}\right] = \frac{\mu_1}{\bar{c}_{44}}\frac{\sqrt{\left(1 - \frac{c^2}{c_1^2}\right)}}{\sqrt{\left(\frac{c^2}{c_0^2} - 1\right)}} \qquad (34)$$

This is the dispersion equation of Love wave in a homogeneous medium over a homogeneous half-space.

7 Numerical computation and discussion

In order to show the effects of dimensionless inhomogeneity parameters and dielectric constant on the propagation of Love-type waves in piezoelectric layer overlying a non-homogeneous half-space, numerical computation of Equations (30) and (33) were performed with different values of parameter representing the above characteristics. For the computational purpose, we represent some numerical data from Nie et al. [23] for piezoelectric layer and from Gubbins [15] for lower non-homogeneous medium in order to study the effect of parameters in the media.
(i) Numerical result for upper piezoelectric layer

Material parameters used in the piezoelectric layer for computation

Material	$c_{44}(\times 10^9 N/m^2)$	$\varepsilon_{11}(\times 10^{-9} C^2/Nm^2)$	$\rho(\times 10^3 kg/m^3)$	$e_{15}(C/m^2)$
PZT-4	25.6	6.45	7.5	12.7

(ii) For the non-homogeneous half-space,

Medium	Rigidity(μ) $(\times 10^{10} N/m^2)$	density(ρ) kg/m^3
Non-homogeneous	7.10	3321

From the above numerical results, the value of $\frac{c_0^2}{c_1^2}$ has been taken fixed to 0.3156 in all the figures. The results are presented in figure 2, figure 3, figure 4 and figure 8 for piezoelectric layer in case of open circuit and figure 5, figure 6, figure 7 and figure 9 in case of electrically short circuit. All the curves have been plotted with vertical axis as dimensionless phase velocity $\frac{c}{c_0}$ against horizontal axis as dimensionless wave number kh. It has been found that the phase velocity decreases as the wave number increases in each of the figures under the assumed values of various parameters.

Figure 2 gives the dispersion curve of Love-type wave against dimensionless wave number in a piezoelectric structure in the case of electrically open circuit. The dispersion curves are plotted for selected values of inhomogeneity factor $\frac{a}{2k}$ and fixed values of dielectric constant ε_{11}. The values of inhomogeneity parameter for curve no.1, curve no.2, curve no.3, curve no.4 and curve no.5 have been taken as $0.0, 0.1, 0.2, 0.3$ and 0.4 respectively. From this figure we have seen that the speed of Love-type waves decreases with the increase of inhomogeneity parameter $\frac{a}{2k}$. Figure 5 shows the effect inhomogeneity in case of electrically short circuit. This figure described that the phase velocity of Love-type waves increases with the increase of inhomogeneity parameter $\frac{a}{2k}$. These curves give the result of the dispersion equation given in particular case I.

In figure 3 and figure 6 attempts have been made to come out with effect of dielectric constant ε_{11} for Love-type wave in both the case of electrically open circuit and short circuit respectively. The curves are plotted for different values of dielectric constant ε_{11} and constant values of inhomogeneity parameter $\frac{a}{2k} = 0.3$. The values of dielectric constant for curve no.1, curve no.2, curve no.3, curve no.4, curve no.5 have been taken as $6.45 \times 10^{-9}, 6.70 \times 10^{-9}, 6.95 \times 10^{-9}, 7.20 \times 10^{-9}, 7.45 \times 10^{-9}$ respectively. From these figures we have seen that phase velocity increases with the increase of dielectric constant ε_{11}.

Figure 4 and Figure 7 represents the dispersion curves in the cases of electrically open and short circuit respectively. The curves are plotted for different values of dielectric constant ε_{11} when the lower half-space is homogeneous that is $\frac{a}{2k} = 0.0$. The values of dielectric constant for curve no.1, curve no.2, curve no.3, curve no.4, curve no.5 have been taken as $6.45 \times 10^{-9}, 6.70 \times 10^{-9}, 6.95 \times$

$10^{-9}, 7.20 \times 10^{-9}, 7.45 \times 10^{-9}$ respectively. The curves of figure 4 concluded that phase velocity increases with the increases of dielectric constant ε_{11}. Whereas from figure 7 we have seen that phase velocity slight increases with the increase of dielectric constant ε_{11}.

Figure 8 represent a screen shot of graphical user interface (GUI) software in MATLAB in the case of piezoelectric open circuit demonstrating the graph plotted in figure 2 as a sample. This GUI generalizes the finding of the present paper by allowing one to vary the ranges of different dimensionless parameters and also by providing different values to the various parameters involved. This will help one to observe the variation on the phase velocity of Love-type wave against dimensionless wave number for different sets of values. Similarly, Figure 9 represent a screen shot of graphical user interface (GUI) software in MATLAB in the case of piezoelectric short circuit demonstrating the graph plotted in figure 5 as a sample.

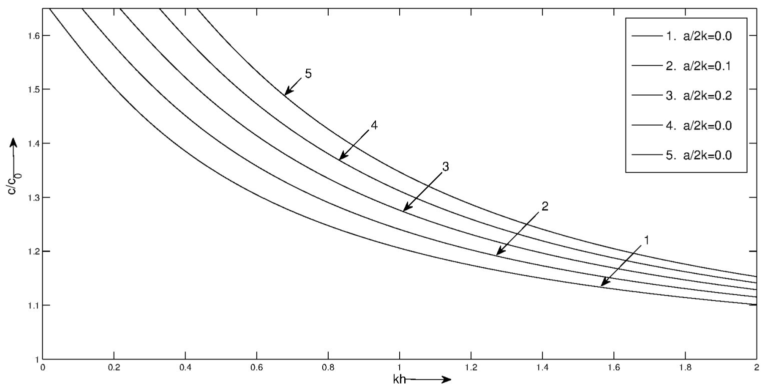

Figure 2: Dimensionless phase velocity $\frac{c}{c_0}$ in PZT-4 system as a function of dimensionless wave number kh of Love-type waves in electrically open case, for fixed value of dielectric constant ε_{11} and different values of $\frac{a}{2k}$.

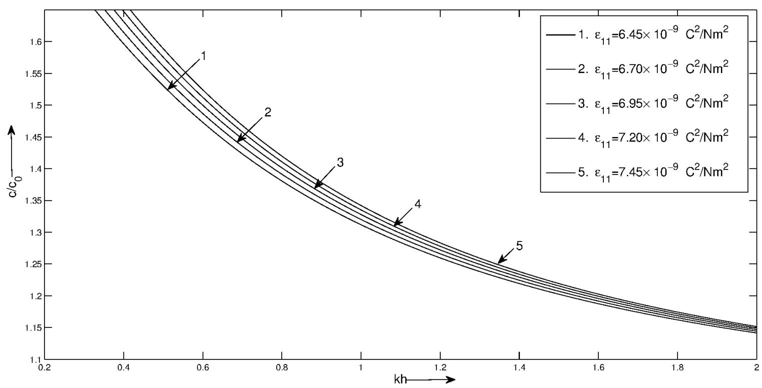

Figure 3: Dimensionless phase velocity $\frac{c}{c_0}$ against dimensionless wave number kh of Love-type waves in electrically open case for constant value of inhomogeneity parameter $\frac{a}{2k}(= 0.3)$ and different values of dielectric constant ε_{11}.

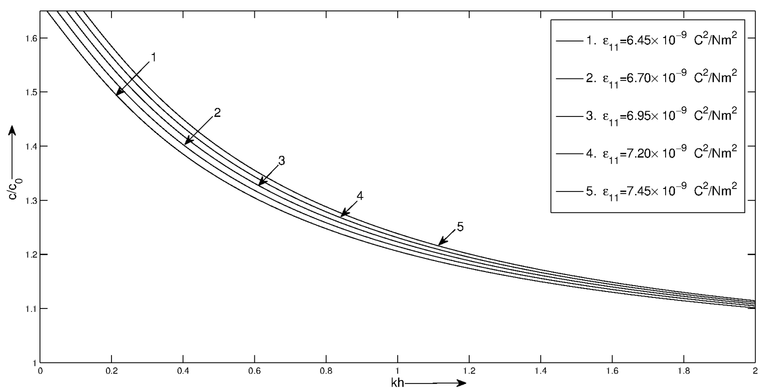

Figure 4: Dimensionless phase velocity $\frac{c}{c_0}$ as function of dimensionless wave number kh of Love-type waves in electrically open case, for different values of dielectric constant ε_{11} in the case of homogeneous half-space.

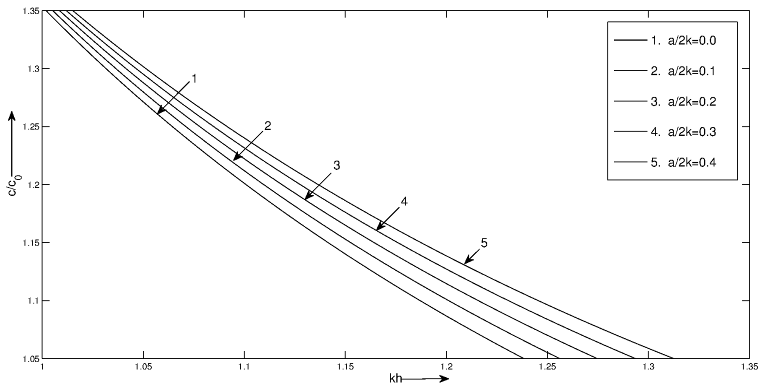

Figure 5: Dimensionless phase velocity $\frac{c}{c_0}$ in PZT-4 system as a function of dimensionless wave number kh of Love-type waves in electrically short case, for fixed value of dielectric constant ε_{11} and different values of $\frac{a}{2k}$.

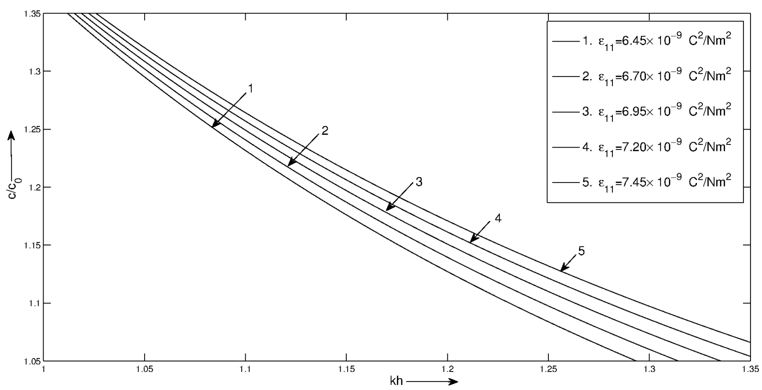

Figure 6: Dimensionless phase velocity $\frac{c}{c_0}$ against dimensionless wave number kh of Love-type waves in electrically short case for constant value of inhomogeneity parameter $\frac{a}{2k}(= 0.3)$ and different values of dielectric constant ε_{11}.

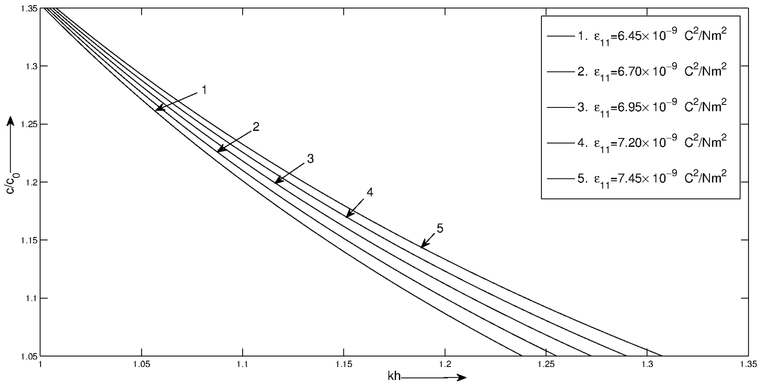

Figure 7: Dimensionless phase velocity $\frac{c}{c_0}$ as function of dimensionless wave number kh of Love-type waves in electrically short circuit case, for different values of dielectric constant ε_{11} in the case of homogeneous half-space.

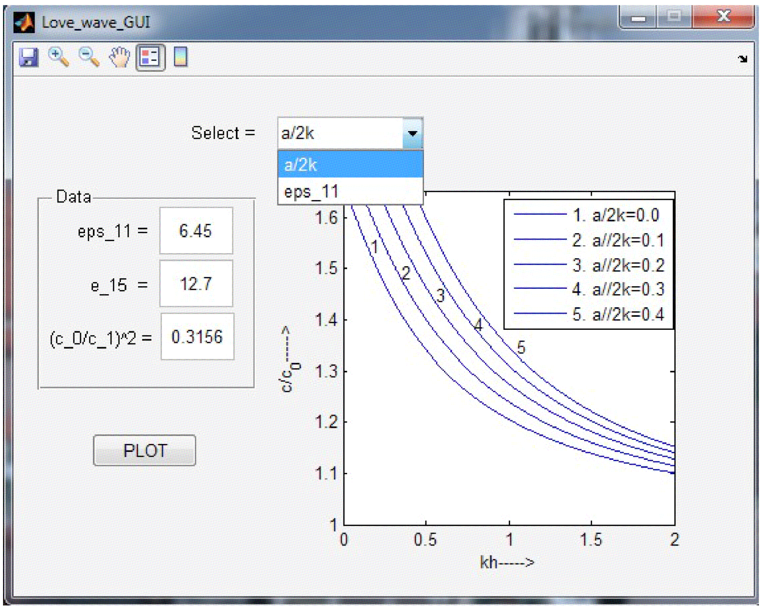

Figure 8: A Graphical user interface (GUI) model showing the variation of dimensionless phase velocity against dimensionless wave number for different values of $\frac{a}{2k}$ in case of electrically open circuit.

8 Conclusions

Propagation of Love-type wave in a piezoelectric layered structure overlying a non-homogeneous half-space has been studied in details. Closed form solutions for the displacements in layer and half-space have been derived separately. Dispersion relations have been obtained separately in the cases of electrical open circuit and short circuit. The effect of inhomogeneity parameter and dielectric constant are shown graphically in both the cases of electrical open and short circuit. From the above figures we may conclude that the phase velocities of Love-type waves are considerably influenced by inhomogeneity parameter and dielectric constant.

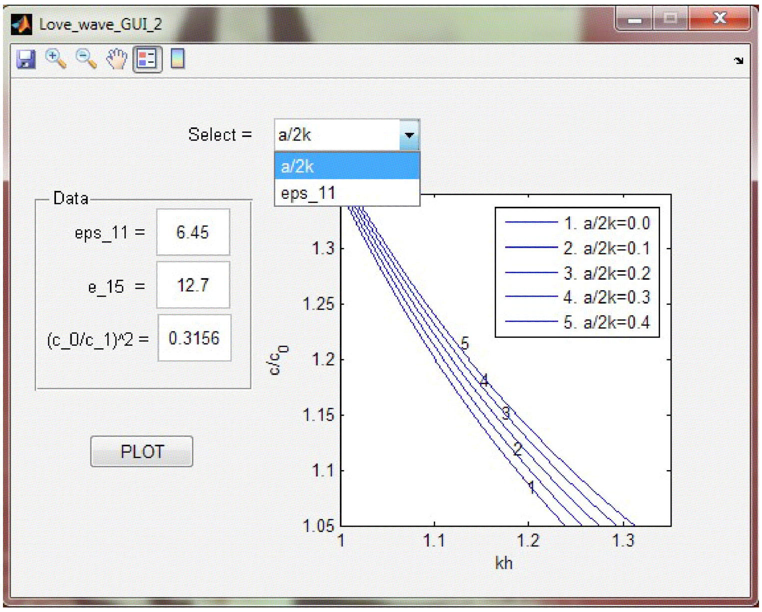

Figure 9: A Graphical user interface (GUI) model showing the variation of dimensionless phase velocity against dimensionless wave number for different values of $\frac{a}{2k}$ in case of electrically short circuit.

In the case of electrically open circuit, we have seen from figure 2 that the phase velocity of Love-type waves decreases with the increases of inhomogeneity parameter. Whereas from figure 3 and figure 4 we have concluded that the phase velocity increases with the increase of dielectric constant. In the case of electrical short circuit, we have seen from figure 5 to figure 7 that the phase velocity increases with the increase of inhomogeneity parameter and dielectric constant.

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